

Why are the points ~~(0,0)~~ and ~~(4.25, 7)~~ not considered local minimum or local maximum points for the cubic function you found in problem 2 to model the Section I design for the roller coaster.

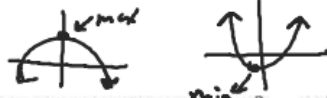
Consider the relationship between the degree of a function and the number of local maximum and/or local minimum points on the graph of a function.

- a. Give an example of a polynomial function with no local maximum or local minimum point.

$$y = mx + b$$

$$y = 4$$

- b. How many local maximum and/or minimum points can there be on the graph of a quadratic polynomial? 1



Maximum
number of extrema
is 1 less than
degree.

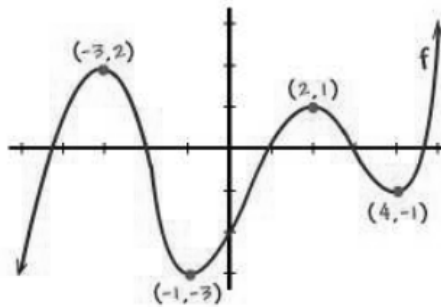
Given the equation, $y = x^3 - 6x^2 + 12x - 3$, how many local max/min would you expect to have? Find them. (2)

Given the equation, $y = x^4 - 9x^2 + 2$, how many local max/min would you expect to have? Find them. (2)

Given the equation, $y = x^5 - 5x^3 + 4x$, how many local max/min would you expect to have? Find them. (24)

What you will learn about:
End Behaviors
Intervals of Increasing/Decreasing

Identify intervals on which the function is decreasing and increasing.



For each function identify the intervals of increasing and decreasing.

$$f(x) = -x^3 + 2x + 2$$

$$f(x) = x^3 - 11x^2 + 39x - 47$$

$$g(x) = \frac{x^2}{4x+4}$$

$$h(t) = \frac{3t^2-3}{t^3}$$

Describe the end behaviors for each function.

$$f(x) = x^3 - 4x^2 + 7$$

$$g(x) = x^4 - 4x^2 - x - 5$$

$$h(t) = t^5 - 4t^3 + 5t + 2$$

$$p(x) = -x^4 + 3x^3 - 5x + 2$$

$$f(x) = -x^3 - 4x^2 + 4$$

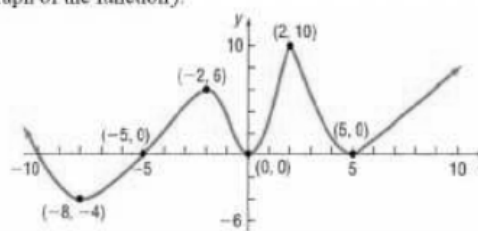
$$f(x) = (x - 3)(x + 5)(x - 1)$$

$$B) f(x) = (x - 3)(5 - 6x)(x - 1)$$

C) $f(x) = (x - 3)^2(x + 5)(x - 1)$

D) $f(x) = (x - 3)(5 - 6x)^3$

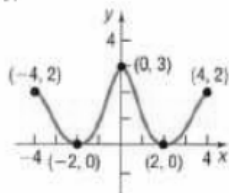
In problems 1-8, use the given graph of the function f .



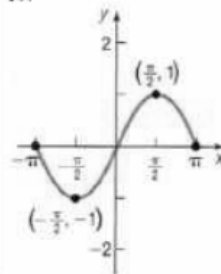
- | | |
|---|---|
| 1 | Is f increasing on the interval $(-8, -2)$? |
| 2 | Is f increasing on the interval $(2, 10)$? |
| 3 | List the interval(s) on which f is increasing. Justify your answer. |
| 4 | List the interval(s) on which f is decreasing. Justify your answer. |
| 5 | List the value(s) of x at which f has a local maximum. Justify your answer. |
| 6 | List the value(s) of x at which f has a local minimum. Justify your answer. |
| 7 | Find the x -intercepts. |
| 8 | Find the y -intercepts. |

- (b) The x - and y - intercepts
- (c) The intervals of increase. Justify.
- (d) The intervals of decrease. Justify.
- (e) The intervals of constant. Justify.

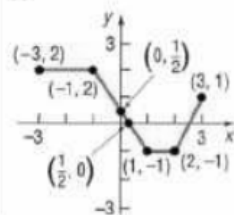
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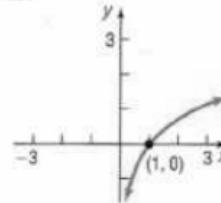
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11.



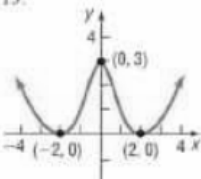
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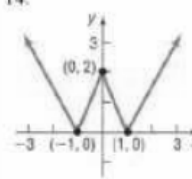
In problems 13-16, the graph of a function f is given. Use the graph to find:

- a) The numbers, if any, at which f has a local maximum. What are those local maxima?
- b) The numbers, if any, at which f has a local minimum. What are those local minima?

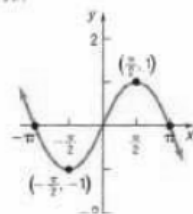
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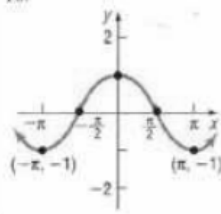
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15.



16.



Addition, Subtraction, and Zeros

When a small music venue books a popular band business prospects of events depend on how the ticket sales are set. For example, if the ticket price at x dollars, income and expenses might be estimated as follows.

Ticket sale income:

$$t(x) = -25x^2 + 750x$$

Snack Bar Income:

$$s(x) = 7,500 - 250x$$

Concert operating costs:

$$c(x) = 4,750 - 125x$$

Snack bar operating costs:

$$b(x) = 2,250 - 75x$$

Before the show the manager uses the functions $t(x)$ and $s(x)$ to estimate total income.

\$ 12 - ticket

- a. What income should the manager expect from ticket sales alone if the ticket price is set at \$12? What income from snack bar sales? What income from the two sources combine?

$$t(12) = -25(12)^2 + 750(12) \\ = 5400$$

$$s(12) = 7500 - 250(12) \\ = 4500$$

$$\text{Total Income} = 5400 + 4500 \\ = 9900$$

- b. What rule would define the function $I(x)$ that shows how combine income from ticket sales and snack bar sales depends on ticket price? Write a rule for $I(x)$ in simplest standard form.

$$I(x) = t(x) + s(x) \\ = -25x^2 + 750x + 7500 - 250x \\ = -25x^2 + 500x + 7500$$

- c. What is the degree of $I(x)$? How does that compare to the degrees of $t(x)$ and $s(x)$?

$$\text{Degree } I(x) = 2$$

Same as highest degree of $t(x) + s(x)$

$$c(x) = 4750 - 125x$$

$$b(x) = 2250 - 75x$$

$$9900 - 4600 = 5300$$

The manager also uses the functions $c(x)$ and $b(x)$ to estimate total operating expenses.

- a. What expenses should the manager expect from concert operations alone if the ticket price is set at \$12? What expense from snack bar operations? What expenses from the two sources combine?

$$c(12) = 4750 - 125(12) = 3250$$

$$b(12) = 2250 - 75(12) = 1350$$

$$\text{Total} = 4600$$

- b. What rule would define the function $E(x)$ that shows how combine expense from concert and snack bar operations depends on ticket price? Write a rule for $E(x)$ that is in simplest standard form.

$$\begin{aligned} E(x) &= c(x) + b(x) \\ &= 4750 - 125x + 2250 - 75x \\ &= -200x + 7000 \end{aligned}$$

- c. What is the degree for $E(x)$? How does that compare to the degrees of $c(x)$ and $b(x)$?

Degree of $E(x)$ is 1

Same as $c(x)$ and $b(x)$

Consider the next function $P(x)$ defined as $P(x) = I(x) - E(x)$.

- a. What does $P(x)$ tell about the business prospects for the music venue? $P(x)$ tells you profit based off

ticket price

- b. Write two equivalent rules for $P(x)$.

- One that shows the separation expressed for income and operating expenses
- Another that is in simplest standard form

$$P(x) = (-25x^2 + 500x + 7500) - (-200x + 7000)$$

$$P(x) = -25x^2 + 700x + 500$$

- c. What is the degree for $P(x)$? How does that compare to the degrees of $I(x)$ and $E(x)$?

Degree of $P(x)$ is 2

Highest of the 2 equations

Find the sum and difference for each set of functions and give the degree in each case. Make sure your answer is in standard form.

$$f(x) + g(x) \quad f(x) - g(x)$$

a. $f(x) = 3x^3 + 5x - 7$ and $g(x) = 4x^3 - 2x^2 + 4x + 3$

b. $f(x) = 3x^3 + 4x^2 + 5$ and $g(x) = -3x^3 - 2x^2 + 5x$

c. $f(x) = x^4 + 5x^3 - 7x + 5$ and $g(x) = 4x^3 - 2x^2 + 5x + 3$

d. $f(x) = 6x^4 + 5x^3 - 7x + 5$ and $g(x) = 6x^4 + 5x$